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when we do. Why does the curve $\rho = \theta^2$ raise the question of the unit of angle measurement while the curve $\rho = \sin \theta$ does not? Is the θ of our polar coördinates a number or an angle? The plotting of the curve $\sin \rho = \theta$ presents the difficulty in even more acute form. The best way out of all these difficulties would seem to be to emphasize the fact that ρ and θ are numbers; and that in polar coördinates ρ is represented graphically by ρ units (any convenient unit) of length, while θ is represented by an angle of θ *radians*.

It is in the course in calculus that the student must be brought to a full realization of the significance of $\sin x$ as a function of a number. In deducing the formula $d \sin x/dx = \cos x$, the idea of x as an angle is still in evidence; but it should be emphasized that the validity of the formula depends upon the fact that x means the number of *radians* in the angle. One good method of impressing this point is *first* to deduce a formula assuming that x means an angle of x degrees, so that the student may see clearly the advantage of the use of the radian unit. Being thus committed to this unit of angle measurement, it will follow later that in the formula

$$\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C,$$

$\sin^{-1} x$ means the number of radians in the angle[1] whose sine is x. How many of our students have any reason for a choice between $\pi/4$ and 45° as a value for the integral $\int_0^1 \sqrt{1 - x^2}dx$, other than the fact that the latter value does not seem to give the "right answer" for the area of the quarter circle?

When we have expanded $\sin x$ in a power series, convergent (happily) for all values of x, we have finally the basis for a definition of the function which is independent of geometric notions. An analogy may be helpful to the student at this point. The geometric notions of the area and length of side of a square may be used to exhibit the relation between the numbers x and $\sqrt{x}$; but nevertheless this number relation is independent of such geometric considerations. And the student of fairly keen mathematical insight will be interested to see that he now has a relation between the numbers x and $\sin x$ which is similarly independent of geometric considerations.

RECENT PUBLICATIONS.

REVIEWS.

MATHEMATICS FOR FRESHMEN.

1. *Introduction to the Elementary Functions.* By R. B. McClenon with the editorial coöperation of W. J. Rusk. Boston, Ginn, 1918. 8vo. 9 + 244 pp. Price $1.80.

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[1] It is also important, of course, that $\sin^{-1} x$ should have been so defined as to make it a single-valued and continuous function.

2. *Freshman Mathematics.* By W. R. Ransom. New York, Longmans, 1918. 12mo. 12 + 285 pp. Price $1.50.
3. *Elementary Mathematical Analysis. A textbook for first-year college students.* By C. S. Slichter. Second edition revised and entirely reset. New York, McGraw-Hill, 1918. 12mo. 18 + 497 pp. Price $2.50.
4. *Unified Mathematics.* By L. C. Karpinski, H. Y. Benedict, and J. W. Calhoun. Boston, Heath, 1918. 12mo. 8 + 522 pp. Price $2.80.

What shall be the content of the course in freshman mathematics? This is a question that has been occupying the minds of many of the teachers of mathematics in our colleges for the past five or six years. The traditional course in mathematics for the freshman year has held sway for a long period of time and the assaults that have been directed against it as a required subject have, in the main, left it secure.

Why should the content and arrangement of the course be changed? The fact that its security might be at stake is neither a sufficient nor an intelligent reason. So long as technique is regarded as the all-important aspect of the subject, there is no reason why there should be any change from the traditional course, except possibly toward unification and even this may be questioned. On the other hand, in view of the fact that freshman mathematics is a required course in the majority of our colleges, there is a feeling that all the emphasis should not be put on technique, but that insight and understanding should receive their due share of emphasis, thus affording a clearer perception or recognition of the worth of the subject in practical application. The four books which we shall now examine attempt to answer the question confronting us.

1. The subject matter which consists of trigonometry, elementary analytic geometry, and an introduction to the differential calculus, is presented in ten chapters (230 pages). This does not include the appendix, containing nine pages of review work in algebra and geometry. The authors take functionality as the unifying principle and endeavor to keep this idea foremost throughout the book.

Chapters I and II deal with graphical representation and functional relations. These two chapters are largely introductory. Much of the material contained in the 32 pages of Chapter III on graphic algebra seems somewhat disconnected and rather elementary. The quadratic equation with which the student should be familiar is taken up in too great detail. It also seems a waste of time and space to devote three pages to the factor theorem for the quadratic equation, and an even greater waste to discuss in this chapter the maximum and minimum values of the quadratic function, which comes up again in the differential calculus. Chapters IV and IX consisting of 52 pages are devoted almost entirely to trigonometry, the solution of triangles by logarithms being deferred until the exponential and logarithmic functions are taken up in Chapter X. This separation of trigonometry into three parts seems rather artificial and causes one to wonder if functionality, which is the unifying principle, does not necessitate an arrangement somewhat unnatural and not conducive to the best results. There seems to be no good reason why Chapters IV and IX should not be combined. In the beginning of Chapter IV the trigonometric functions are defined at once for any angle. In this chapter the work is well arranged. Although velocities and forces are considered under the application of the trigonometric functions, radian measure, and angular velocity are nowhere mentioned.

Chapter V deals with simple irrational functions and the locus problem, and the three following chapters are devoted to the consideration of the straight line, the circle and the conics. In Chapters VI and VII the geometric aspect of the work receives emphasis, while graphic algebra predominates in Chapter VIII.

Some disappointment is felt over the fact that in Chapter X, the logarithmic and exponential functions seem to be introduced solely for the use of logarithms in the solution of triangles. In our opinion not enough consideration is given to the functional relation $y = \log x$. The important functions $y = \log_e x$, $y = e^x$ and $y = e^{-x^2}$ are not mentioned. The concluding chapter of the book, which is on the differential calculus, presents in 32 pages, some necessary theorems on limits, the definition of the derivative of a function, rules for finding the derivative of some of the simpler functions, rates, and maxima and minima. There is also a well-chosen set of problems. The material of this chapter is well selected and clearly presented. In our opinion the notation for the derivative, D_xy, is most confusing to the beginner and for this reason should not be employed.

2. If the object of the freshman course in mathematics is to give the student interesting material with which to work and from which may be derived useful information, and if mathematical coherence and completeness are of no great importance to the freshman, except as demanded for such information, this book can be highly praised. But the well-selected material is not the only merit the book has. The work is well presented and the explanations are clear. It is quite surprising too, to find that so much can be put into 285 pages without undue congestion.

There are twenty-six short chapters in the book. The more elementary part of the work is given in the first seven chapters. The leading topics of these chapters are arithmetical computation, the application of trigonometry to right triangles, and an introduction to coördinates. The trigonometric functions are defined for acute angles and the definitions for any angle are not given when coördinates are first introduced but are deferred until the consideration of vectors comes up (Chapter XII) toward the close of the work on trigonometry. This causes an awkward situation on page 96 where the relation, $\sin A = \sin (180° - A)$, must be given as a definition.

The next seven chapters of the book are devoted entirely to trigonometry and its applications. The use of the slide rule is explained and a chapter on spherical trigonometry and its applications is added.

Determinants, variation, and the solution of numerical equations are the topics of algebra which are taken up in Chapters XIV–XVII.

The last eight chapters of the book deal primarily with differential and integral calculus and their applications. The derivative is made use of in plotting exponential and other special curves; in the study of rates and in finding maximum and minimum values. The fifteen pages of the integral calculus are devoted to its application in finding areas and volumes.

The book on the whole is interesting and stimulating. The exercises and problems have been selected and graded with care, and no demand is made on the students' ability to handle complicated algebraic expressions. On the other hand, the book lacks coherence. The analytic geometry that is given is scattered throughout the book, and one has a feeling that this very important subject has not received the attention it merits. The conic sections are inadequately treated and many of their important properties are not mentioned. Since one of the most important problems that confronts the author of a unified course in mathematics, is the proper selection from a wealth of material, it seems unwise to devote twelve pages to spherical trigonometry and deal scantily with analytic geometry.

3. Despite the author's assertion in the preface that "this book is not intended to be a text on 'Practical Mathematics,' in the sense of making use of scientific material and of fundamental notions not already in the possession of the student, or in the sense of making the principles of mathematics secondary to technique," the book has the appearance of a manual on practical mathematics.

On the first reading one is inclined to say that the book has been written from the engineering point of view. But while the author deals elaborately with the practical applications of all of the mathematical material that the student has on hand or acquires throughout the course, he none the less lays emphasis on the proper understanding of mathematical principles and, as regards technique and application, strives to maintain the proper balance. Just how well he has succeeded in doing this it is hard to say.

The book will certainly arouse interest and provoke thought. It shows breadth of thought both in conception and execution. It is impossible to pass judgment, without a fair trial, on its usefulness as a textbook.

The first three chapters of the book deal with scales, rectangular coördinates, variables and functions, and the power function. In our opinion this part of the work is given in too great detail. The author has an admirable way of presenting transformations early in the course. Certain theorems on loci are stated in connection with the graphs of functions and these theorems explain in a simple but satisfactory way translation, rotation, expansion and shear.

In Chapter IV the trigonometric functions are defined for any angle and the treatment of the functions is taken up in connection with the analytic geometry of the circle. While Chapter V, on the ellipse and hyperbola has some connection with Chapter IV, there is a rather abrupt change in passing to the selected topics of algebra in Chapters VI–VIII. After this there is another change in taking up the logarithmic and exponential functions of Chapter IX and the remainder of the work in trigonometry in Chapter X. A better arrangement seems possible. Why can not Chapter IX follow Chapter III and Chapter X follow Chapter IV? This seems more desirable and more natural from the point of view of coherence. The functionality idea would thus be predominant in the first four chapters and there would be a natural shift through the circular functions to the two following chapters in trigonometry.

A review of the most important topics of secondary school algebra is found in the twenty-four pages of the appendix.

4. To what extent have the authors succeeded in unifying algebra, trigonometry and analytic geometry and what is the process of unification? They have not chosen the way of functionality. The notion of a function first appears on page 58, and while it is constantly in use throughout the book, it is not predominant and does not determine the arrangement of the subject matter.

The part of the book that has been unified in a splendid way is the first seventeen chapters (280 pages). This includes the application of algebra to arithmetic, trigonometry and its applications, and the more elementary part of analytic geometry. The graphical method serves in a most natural way as a connecting link. A characteristic feature of this part of the book is the "timing exercises." Wherever a set of exercises is given for the purpose of developing the students' mechanical ability, a tentative time is specified. This will undoubtedly stimulate interest and challenge the students' mental activity. The next seven chapters are devoted to the conics and their applications. The most important properties of the conics are derived and the subject is enlivened by a splendid collection of practical problems. In connection with the practical problems on elliptic and parabolic arches, there are several photographs of bridges where these arches are used. There are also photographs showing the use of elliptic gears.

The solution of numerical equations, wave motion, laws of growth, polar coördinates, complex numbers, and solid analytic geometry are the subjects considered in the last eight chapters. The two most interesting chapters in this part of the book are those on wave motion and laws of growth. The authors have succeeded splendidly in presenting, in connection with the more formal part of the work, interesting problems that have practical application. Indeed this is a distinctive feature of their book. The problem about the big gun that bombarded Paris and the curve of healing of a wound, give evidence that they are up-to-date. While they have placed much emphasis on the practical application of the subject matter, they have not neglected the more formal part of the work.

There is some doubt as to how far they have succeeded in unifying the later material. The latter part of the book seems to lack the coherence that is characteristic of the first part. Perhaps this is due to the fact that trigonometry forms the body of the first part.

There are many historical notes throughout the book.

The regularity of the type gives it an attractive appearance.

The following tables are bound in with the text: squares, cubes, square roots, cube roots, and reciprocals (from 1 to 100); logarithms; natural functions; radian measure of angles; tables for e^x and e^{-x}; tables of interest functions.

Having examined these four books in detail let us now take a broader view of them.

So far as unification is concerned, they have a common purpose, though Ransom does not give as much attention to coherence as the others do. There are differences however in the subject matter as well as in the way by which unity may be accomplished. There is also a certain difference in the spirit of the books. The chief purpose of McClenon and Rusk seems to be to unify the subject matter with which they are concerned. The other three books breathe a different spirit. They, too, seek coherence but, in addition to this, they agree that a prominent feature of the work of the freshmen, should be the use of interesting and helpful problem material—material that brings together the workaday world and mathematical principles. They have succeeded admirably in collecting such material.

As regards subject matter, there is rather close agreement on the work in trigonometry. Has enough attention been given to trigonometry or has it been cut down to an irreducible minimum? In McClenon there are 73 pages on trigonometry and its application; in Ransom, 76 pages; in Slichter, 90 pages; in Karpinski, Benedict and Calhoun 114 pages. In our opinion the best arrangement of the trigonometry is given by Karpinski, Benedict and Calhoun. Ransom scatters the subject throughout the book in six different parts.

There is disagreement about the topics in college algebra. McClenon and Rusk confine their attention to graphic algebra. Ransom devotes 32 pages to determinants and the solution of numerical equations; Slichter gives 17 pages to permutations and combinations; Karpinski, Benedict and Calhoun give 14 pages to the solution of numerical equations.

Analytic geometry with its applications is given far greater emphasis by Karpinski, Benedict and Calhoun, and Slichter than by the other two authors. Since Slichter and Karpinski lay so much emphasis on the practical application of mathematics, it is rather surprising that they do not give the fundamental ideas of the differential and integral calculus, and their applications to the simplest functions. In Karpinski the work on poles and polars, transformations, polar coördinates, complex numbers, and solid analytic geometry covers about 90 pages. In our opinion it would be more profitable to devote this space to the calculus. Slichter could also find time for the calculus by not going into such elaborate detail in different parts of the book. Many of his directions, specifications and remarks could be left to the teacher.

The tendency to make as wide use as possible, of practical problems in illustrating mathematical principles is a step in the right direction. When the teachers of mathematics know more of its applications and make wider use of them, and when the teachers of allied subjects call into use the mathematics that the student may have in his possession, there will then be a greater recognition of its supreme importance.

It means little to praise or criticize these books without first giving them a fair trial. Whatever their faults, this new arrangement of material should be tested in the class room. We welcome the books as a contribution towards the solution of our problem. They will be of service to every mathematics teacher of college freshmen.

GEORGE W. MULLINS.